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ON THE PARAMETERIZATION OF MAXIMAL AND CRITICAL CHARACTERISTICS FROM CONTINUOUS POINT SOURCE AT DIFFERENT METEOROLOGICAL CONDITIONS

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ABSTRACT

On the basis of generalization of Ragland's (1976) method, it is determined the maximal and critical (worst-case) pollutant parameters from Gaussian dispersion plume model. It is used Brigg's formula for effective height and dispersions σ_z, σ_y, which are approximated with power laws.

It is considered the effects of gravity deposition, inversion and the dependence of the wind speed on the height. At these conditions general analytical expressions are derived for urban and rural regions for the critical concentration and its location, critical wind speed at ten meters height, the "minimal" stack height for newly planned industrial sources, etc. The meteorological conditions are parameterized in terms of the Pasquill-Turner stability classes A-F.

It is considered and some of these problems with application to non-Gaussian diffusion model.

The result can be used to estimate and standardize the worst-case ambient air concentrations from continuous sources and for determination of the meteorological conditions at which they occur.

Key Words: Critical Parameters, Stability Classes, Deposition, Inversion, Gaussian and non-Gaussian Model.

1. INTRODUCTION

The maximal concentration obtained from the stack of a given industrial region depends mainly on the stack parameters and the meteorological conditions. To find the critical (maximal of the maximal) concentration, the distance, wind speed and the stratification conditions under which it occurs is more important task. The critical parameters allow to estimate the worst-case ambient pollution conditions, to determinate the stack height of newly planned industrial sources, and also for evaluation of the environmental impact of already existing sources.

Basic method for determination of the critical parameters at power laws for the dispersions and constant with the height speed gives Raglang (1976). Here we will extend the application of this approach considering some more complex diffusion and meteorogical conditions and actualized data for the dispersions and the effective height of the source. On the basis of power law approximation of the well known dispersion curves of Briggs for $\sigma_z(x)$ and $\sigma_y(x)$, it appeared to be possible to obtain analytical expressions for the critical parameters for a relatively wide range of diffusion and meteorological conditions.

2. FORMULATION OF THE PROBLEM

Let's use the Gaussian pollution distribution formula, from source situated in the point $x = y = 0$. The ground level ($z = 0$), concentration C along the plume centerline ($y = 0$) is given by:

$$C = \frac{Q}{\pi U_H \sigma_y \sigma_z} \exp\left[- H^2 / 2\sigma_z\right]$$

(1)

where Q is emission rate, U_H is the wind speed at the effective stack height H. The quantity H is calculated according Briggs formula (see Hanna, 1982).

$$H = h_s + \Delta h \,, \Delta h = FU_s^{-l} \,,$$

(2)

where h_s is the geometric stack height, Δh is the plume rise, $U_s = U(z = h_s)$, F is characteristic technological parameter, l is parameter with value 1or1/3 (see table.1). Wind profile $U(z)$ is given as:

$$U(z) = U_{10}(z/10)^m$$

(3)

where U_{10} is the wind speed at standard level 10m and the parameter m depends on the Pasquill stability classes (Hanna, 1982), see table.1.

Taking into account (3) we receive the following expressions for U_H and U_s:

$$U_H \equiv U(z = H) = U_s \left(\frac{H}{h_s} \right)^m , U_s \equiv U(z = h_s) = U_{10} \left(\frac{h_s}{10} \right)^m$$

(4)

For dispersion parameters $\sigma_z(x)$, $\sigma_y(x)$, it is used the well known formulas of Briggs. In the present work they are approximated with enough precision with the convenient for work power laws:

$$\sigma_z(x) = ax^b \,, \sigma_y(x) = cx^d \,,$$

(5)

where the approximation coefficients a, b, c, d are given in table 1.

Table1. Values of the used parameters for calculation for rural and urban regions.

Z_0=0.03m - Rural							Z_0=1m - Urban					
Kl.	m	a	b	c	d	l	m	a	b	c	d	l
A	0,17	0,20	1,00	0,36	0,92	1	0,06	0,08	1,15	1,42	0,76	1
B	0,175	0,12	1,00	0,34	O,89	1	0,07					
C	0,2	0,30	0,79	0,25	0,87	1	0,075	0,20	1,00	1,32	0,72	1
D	0,27	0,76	0,57	0,20	0,86	1	0,13	0,91	0,72	1,14	0,70	1
E	0,39	1,04	0,47	0,26	0,80	1/3	0,33	0,93	0,69	0,87	0,69	1/3
F	0,61	1,15	0,39	0,34	0,73	1/3	0,54					

2.1 Maximal parameters and normalization procedure

Differentiating (1) by x, taking into account (5), and nullifying the obtained expression, leads to the following relation for distance x_m at which the surface concentration has maximum(Ragland, 1976):

$$x_m = k_1 H^{1/b}, k_1 = \left[\frac{b}{a^2(b+d)}\right]^{1/2b}$$

(6)

The dependence of x_m on H (taking into account the parameters of table.1.) is demonstrated in Fig.1. Inserting (6) in (1), taking into account (4) and (5), we define the maximal surface concentration C_m :

$$C_m = \frac{Q}{\pi U_{10}} \frac{k_2 10^m}{k_1^{b+d}} H^{-\left(m+\frac{b+d}{b}\right)}, \qquad k_2 = \frac{\exp((b+d)/2b)}{ac}$$

(7)

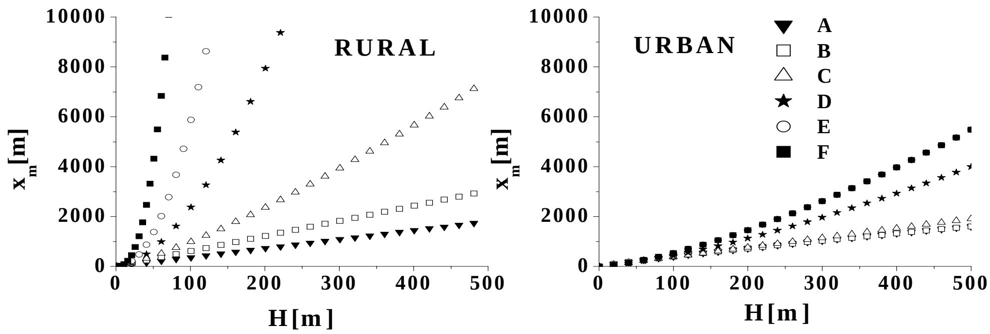

Figure 1. The dependence of x_m on H at different stability classes for rural and urban region.

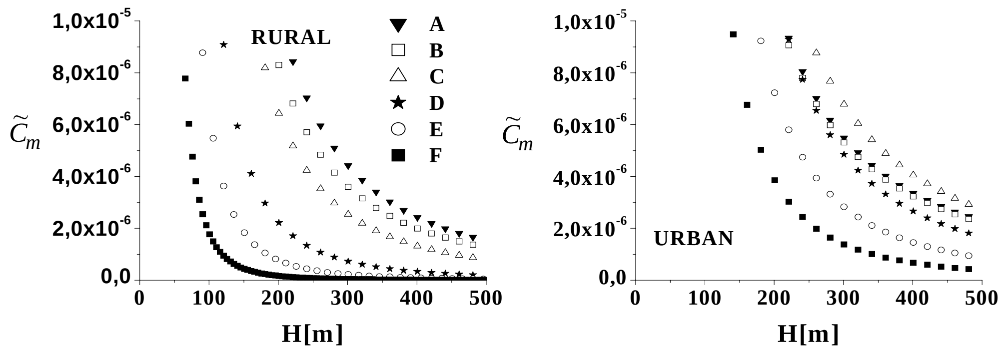

Figure 2. Same as in Fig. 1, but for $\widetilde{C}_m$.

On Fig.2 is demonstrated the dependence of $\widetilde{C}_m = C_m/(Q/\pi U_{10})$ on H for rural and urban region.. It can be seen that $\widetilde{C}_m$ is greater for urban region because of the more intensive mixing turbulent processes over it. Using the so determined maximal parameters x_m and C_m and considering (5), the surface concentration (1) can be normalized:

$$C_s = \frac{\exp(2r)}{\widetilde{x}^{4r}} \exp\left[-\frac{2r}{\widetilde{x}^2}\right],$$

where $C_s = C/C_m$, $\widetilde{x} = (x/x_m)^b$, $r = (1/2)(1 + d/b)$. The dependence of C_s on $\widetilde{x}$ for different stability classes is presented on Fig.3

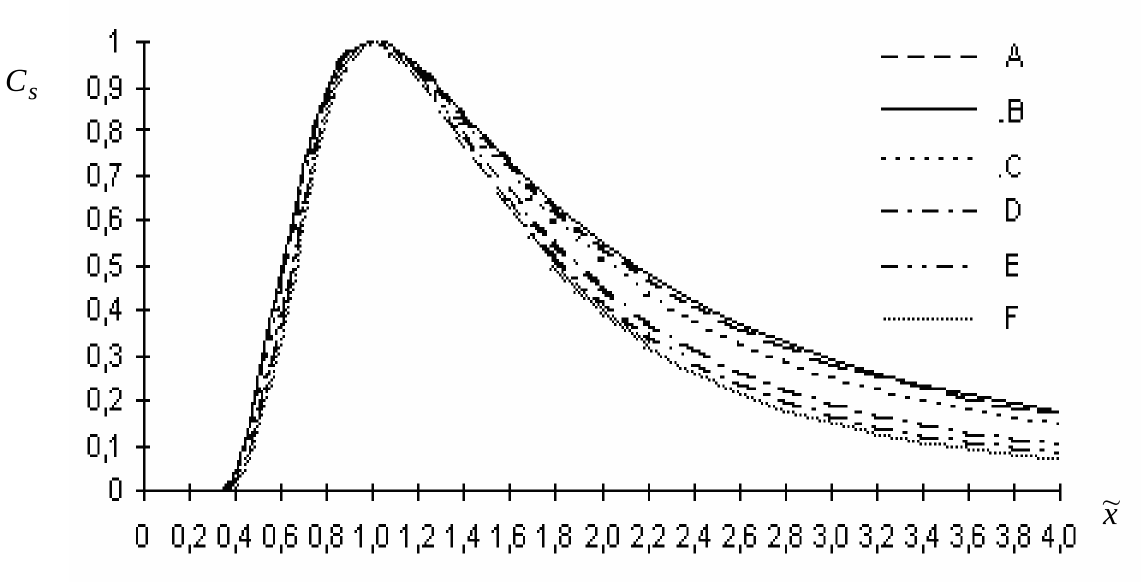

Figure 3. Dependence of the normalized surface concentration $C_s = C/C_m$ on dimensionless distance $\widetilde{x} = (x/x_m)^b$ at different stability classes

2.2 Critical parameters

A great practical interest is the finding of a critical value U_{10cr}, at which for all meteorological conditions, the concentration C_m is maximized. Taking into account (2), (4) and differentiating C_m from (7) by U_{10} and setting the obtained expression equal of zero (condition for extremum), the following expression of critical wind velocity value U_{10cr} can be obtained:

$$U_{10cr} = F^{1/l} 10^m k_3^{1/l} h_s^{-\frac{1+ml}{l}}.,$$
(8)

where $k_3 = l(m+k)-1, k = 1+\dfrac{d}{b}$.

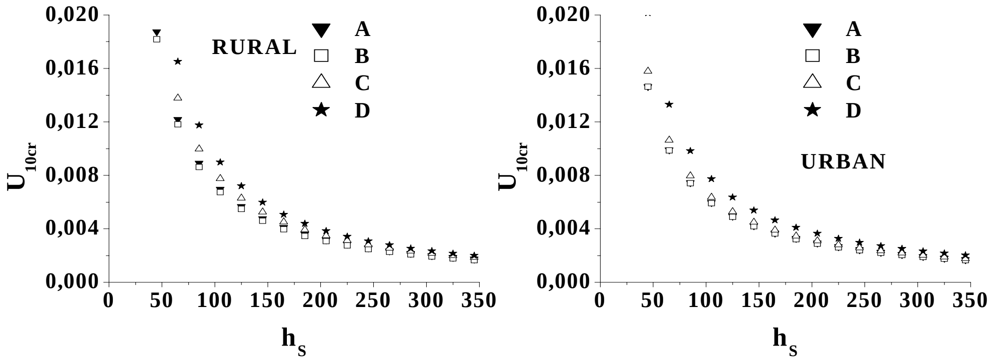

Figure 4. The dependence of the critical speed U_{10cr} on h_s at different stability classes for rural and urban region.

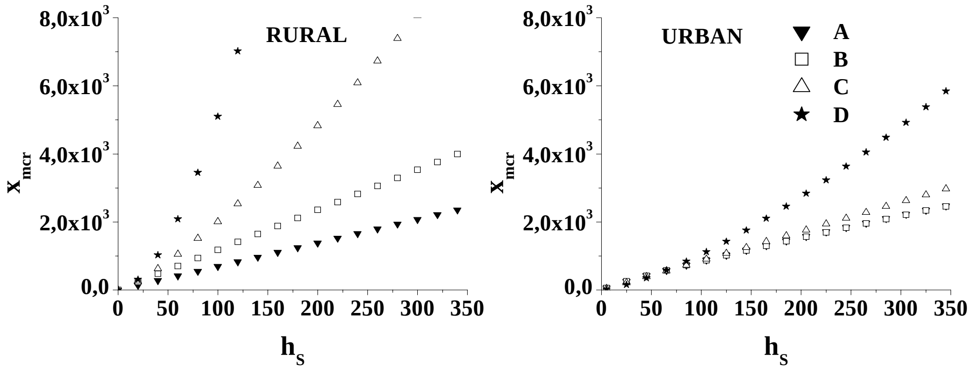

Figure 5. Same as Fig.4, but for x_{mcr}.

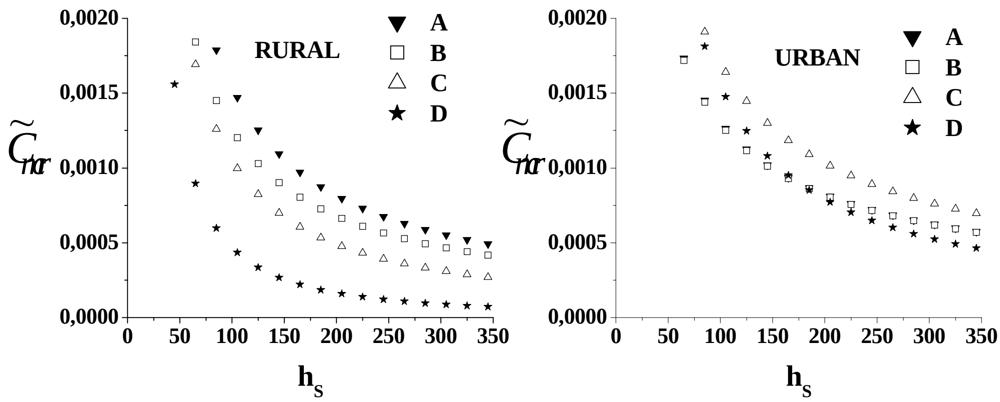

Figure 6. Same as Fig.4, but for $\widetilde{C}_{mcr}$.

Now substituting (8) in (6), taking into account (4), we obtain the critical distance x_{mcr} at which C_{mcr} is realized:

$$x_{mcr} = k_1\left(1+\frac{1}{k_3}\right)^{1/b} h_s^{1/b}.$$

(9)

Considering (8), (9), (2), (4) from (7), we determine the critical (maximal of the maximal) surface concentration C_{mcr}:

$$C_{mcr} = \frac{Q}{\pi}\frac{k_2}{k_1^{b+d}}\left(\frac{h_s}{F}\right)^{1/l} h_s^{-\frac{b+d}{b}}\left(1+\frac{1}{k_3}\right)^{-m-\frac{b+d}{b}} k_3^{-1/l}$$

(10)

On Fig.4-6 are presented the dependence of critical parameters U_{10cr}, x_{mcr} and $\widetilde{C}_{mcr}$ on h_s at selected stability classes A, B, C, D at which it is more likely to form critical condition for high stack sources.

Let's now determine the so called stack height of a planned new source h_{sp} so, that at any meteorological conditions the surface pollution concentration does not exceed the Limit Admissible Concentration (LAC)-C_{LAC} (for given pollutant.) i.e. $C_{mcr} = C_{LAC}$:

$$h_{sp} = \left[C_{LAC}\frac{\pi}{Q}F^{1/l}\frac{k_1^{b+d}}{k_2}\left(1+\frac{1}{k_3}\right)^{m+\frac{b+d}{b}}\right]^{\frac{bl}{b-bl-bd}}$$

(11)

Here we will give only qualitative analyze of some calculated critical parameters (because of the limit space) for high ($h_s \in 120-320m$), strong overheated ($\Delta T_s \in 100-160K^o$), with diameter ($D \in 5-12m$), emission speed. $(V_s \in 6-20\,m/s)$ and emission of SO_2 ($0.1-10\,kg/s$) sources. The respective critical parameter are in the range $U_{10cr} \in 4-9\,m/s$, corresponding for classes $C-D$, $C_{mcr} \in (1.3-4.5)C_{LAC}$ and $x_{mcr} \in 4-40km$.

2.3 Effect of gravity deposition

In the case with gravity deposition velocity w_0, formula (1) turns to the form (see Wark and Warner 1976):

$$C = \frac{Q}{\pi U_H \sigma_y \sigma_z}\exp\left[-(H-\widetilde{w}_0 x)^2/2\sigma_z\right]$$

(12)

where $\widetilde{w}_0 = w_0/\overline{U}$, $\overline{U} = (1/H)\int_0^H U(z)dz = \frac{U_{10}}{1+m}\left(\frac{H}{10}\right)^m$

Applying similar procedure as at the determination of (6) (at $w_0 = 0$), but taking into account (12), now we obtain the following algebraic equations for determination of x_{mw} in the case of considering gravity deposition:

$$a^2(b+d)x_{mw}^{2b} + \tilde{w}_0(2b-1)Hx_{mw} + \tilde{w}_0^2(1-b)x_{mw}^2 - bH^2 = 0 .$$

(13)

Equation (13) can be easily numerically integrated. Here we will limit to the cases $b=1$ and $b=1/2$, at which (13) becomes a quadratic equation which have analytical solution. We will note that most of the values of b are in the range 0.5-1. At $b=1/2$ (around that value approximately is related the stability classes D, E, F at rural region) the solution of (13) is:

$$x_{1mw} = x_{1m}\varphi_1 , \varphi_1 = \frac{2}{1+\sqrt{1+4\tilde{w}_0 px_m}} , \qquad p = 4a^2/(1+2d)$$

(14)

where $x1_m$ is given with (6) at $b=1/2$. Obviously at $w_0 = 0$ (14) coincide with (6).

At $b=1$ (the classes A, B, C approximately unites around this value), the respective solution of (13) is:.

$$x_{2mw} = x_{2m}\varphi_2 , \qquad \varphi_2 = \frac{1}{\tilde{w}_0 / 2a(1+d)^{1/2} + \sqrt{\left(\tilde{w}_0 / 2a(1+d)^{1/2}\right)^2 + 1}} ,$$

(15)

where x_{2m} is given with (6) at $b=1$.

Taking into account (14), (15) identically to the determination of (7) at $w_0 = 0$, we find now:

$$C_{1mw} = C_{1m} f_1 , \qquad f_1 = \frac{1}{\varphi_1^{L_1}} e^{-L_1(R_1-\varphi_1)/\varphi_1} , \qquad R_1 = \left[1 - \frac{\tilde{w}_0 H\varphi_1}{a^2(1+2d)}\right]^2 , \qquad L_1 = \frac{1+2d}{2}$$

(16)

$$C_{2mw} = C_{2m} f_2 , \qquad f_2 = \frac{1}{\varphi_2^{1+d}} e^{-L_2\left(R_2-\varphi_2^2\right)/\varphi_2^2} , R_2 = \left[1 - \frac{\tilde{w}_0\varphi_2}{a(1+d)^{1/2}}\right]^2 , L_2 = \frac{1+d}{2} ,$$

(17)

where C_{1m} and C_{2m} are given by (7) at $b=1/2$ and $b=1$ respectively.

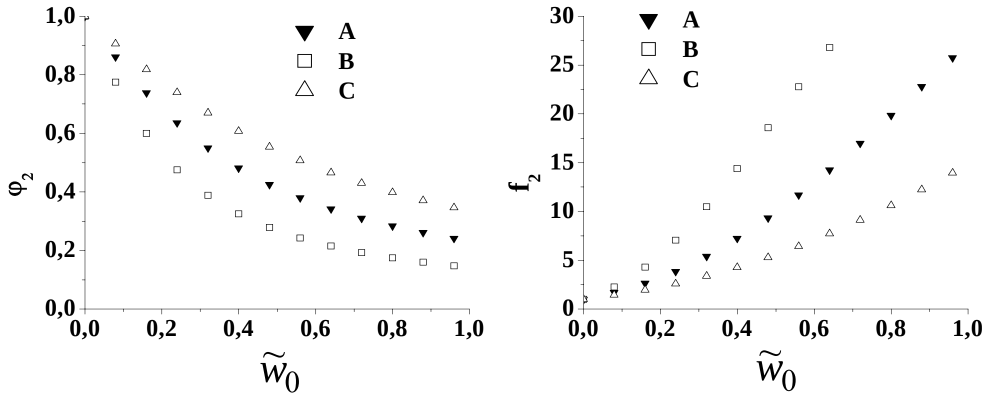

Figure 7. Dependence of correction deposition functions $\varphi_2 = x_{2mw}/x_{2m}$ and $f_2 = C_{2mw}/C_{2m}$ on $\tilde{w}_0$ at different stability classes A, B, C for rural region.

As it can be seen from (14)-(17) the maximal characteristics x_{mw} and C_{mw} are expressed with the respective values for the case $\tilde{w}_0 = 0$ and corrections functions considering the gravity deposition. So for example the dependence of the deposition correction functions φ_2 and f_2 on the parameter $\tilde{w}_0$ and the stability classes is shown on Fig 7. From this figure it can be seen the different degree of decreasing of x_{2mw} and respective increasing of C_{2mw} depending on the gravity deposition parameter $\tilde{w}_0$ at stability classes A, B, C.

2.4 Inversion effect

In the inversion case with gravity deposition, the ground level concentration along the plume center line is given by:

$$C = \frac{Q}{\pi \sigma_y \sigma_z U_H} \sum_{n=-\infty}^{\infty} \exp\left[-\frac{\left(\tilde{H} - 2nh_I\right)^2}{2\sigma_z^2} \right],$$

(18)

where $\tilde{H} = H - \tilde{w}_0 x$, h_I is the inversion height. The worst case will occur when the plume rise is just up to the inversion layer ($H \equiv h_I$). If we take only the first two main terms of the series we receive for the critical concentration the expression:

$$C_{cr} = \frac{Q}{\pi \sigma_y \sigma_z U_H} \left\{ \exp\left[-\left(H - \tilde{w}_0 x\right)^2 / 2\sigma_z^2 \right] + \exp\left[-\left(H + \tilde{w}_0 x\right)^2 / 2\sigma_z^2 \right] \right\}$$

(19)

In the private case $w_0 = 0$, from (19) follows the received by Ragland (1976) formula. The consideration of the gravity deposition makes more complex the problem for determination of x_{mcr} and C_{mcr}. In this case for the determination of x_{mcr} we have an equation of type like (13), which can be solve numerically.

2.5 On the non-Gaussian model application

If we represent the solution of the steady-state diffusion equation for continuous point source in PBL:

$$U(z)\frac{\partial C}{\partial x}+V(z)\frac{\partial C}{\partial y}=\frac{\partial}{\partial z}\left(k_z(z)\frac{\partial C}{\partial z}\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial C}{\partial y}\right)$$

(20)

in the form $C=C_1(x,z)C_2(x,y)$, taking into account Taylor's frozen turbulence hypothesis: $k_y=(1/2)U(z)\left(d\sigma_y^2/dx\right)$, we obtain (Syrakov, Stefanova 2001):

$$C=C_1(x,z)C_2(x,y), \qquad C_2=\frac{1}{\sqrt{2\pi}\sigma_y}\exp\left[-\frac{(y-Rx)^2}{\sigma_z^2}\right],$$

(21)

where C_1 describes the vertical (from linear source at the axis Oy), and C_2 - the horizontal diffusion. The parameter R, determined according to the condition imposed by the splitting of the problem (20): $V(z)/U(z)$~const, is:

$$R=\frac{\overline{V}}{\overline{U}}=\frac{\sin\alpha+C_d^2\cos\alpha}{\cos\alpha-C_d^2\sin\alpha}, \qquad (22)$$

where $\overline{U}$, $\overline{V}$ are the averaged with the height in PBL components of the wind velocity, received by considering the equations of motion, $C_d=U_*/G_0$ is the geostrophic resistance coefficient, α - angle of full turning of the wind in PBL (between the surface and geostrophic wind). At $R=0(\overline{V}=0)$ follows the well known Gaussian distribution for C_2 perpendicular to the wind velocity. At $R\neq 0$ the integral average turning of the wind ($\overline{\alpha}=arctgR$) in the whole PBL is considered in C_2. On Fig.8 is presented the parameterized, by the Pasquill stability classes, integral turning parameter R (Syrakov, Stefanova 2001). It can be seen that with increasing of atmospheric stability the integral effect of turning of the wind in PBL also increases.

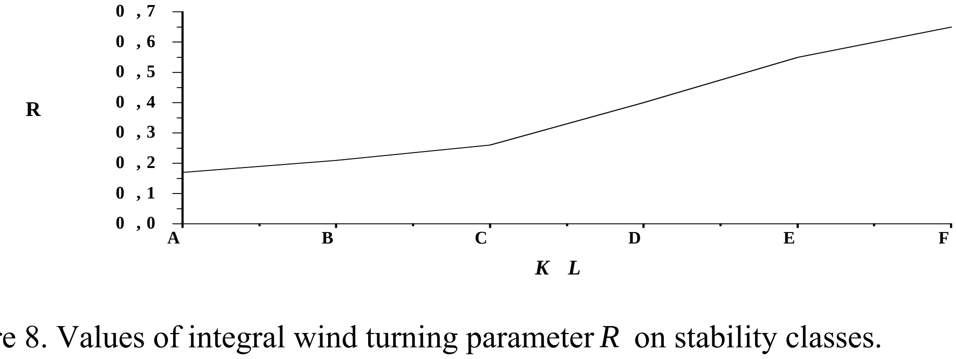

Figure 8. Values of integral wind turning parameter R on stability classes.

Let's now see the vertical diffusion C_1. At power laws for $U(z)$ and $k_z(z)$:
$U(z)=a_u z^m, k_z=b_k z^n$,
(23)
Huang (1979) received the following analytical solution:
$$C_1=Q\exp\left[-\frac{a_u\left(z^\alpha+H^\alpha\right)}{b_k\alpha^2 x}\right]\frac{(zH)(1-n)/2}{b_k\alpha x}I_{-\nu}\left[\frac{2a_u(zH)^{\alpha/2}}{b_k\alpha^2 x}\right],$$
(24)
where $\alpha=2+m-n$, $\nu=(1-n)/\alpha$, $I_{-\nu}$ is a modified Bessel function of first kind of order $(-\nu)$, Q is the source strength, H is effective height given by (2). This solution is detailly explored and used for determination of the statistical moments in (Brown et al,. 1997). From the non-Gaussian solution (21), (24), at $z=0, y=0$ and $R=0$ we obtain respective expression for the surface, along the plume axis, concentration. Considering that $a_u=U_{10}10^{-m}$ (from the comparison of (23) and (3)) follows that this concentration depends on U_{10}. From the condition for extremum about U_{10} and using a procedure identical to that in paragraph (2.2), now we can receive analytical expression for all critical parameters (8)-(11). For example for U_{10cr} and X_{mcr}, we have:
$U_{10cr}=B^{1/l}10^m M^{1/l}h_s^{-(ml+1)/l}$,
(25)
$$x_{mcr}=\frac{1}{b_k\alpha^2(1-\nu+d)}B^{1/l}\left(1+\frac{1}{M}\right)^\alpha h_s^{\frac{(2-n)l-1}{l}},$$
(26)
where $M=l[m+k_2/k_1]-1$, $k_1=-(1+d)$, $k_2=d(n-2)-1$. These parameters are modification of (8) and (9) for the case when it is used the non-Gaussian plum model. The difference is that instead of $\sigma_z(x)$ in the non-Gaussian model it's used k_z-closure. Besides that and at the non-Gaussian model, we can determine the respective dispersion $\sigma_z(x)$, using the definition formula $\int_0^\infty(z-\bar{z})^2\,\overline{C}(x,z)dz\Big/\int_0^\infty\overline{C}(x,z)dz$, where $\overline{C}(x,z)$ is the integrated by y concentration C. For surface source ($H=0$) and $R=0$ for σ_z we have:
$$\sigma_z(x,H=0)=\frac{\Gamma(3/2)}{\Gamma(1/2)}\left(\frac{b_k\alpha^{2x}}{a_u}\right)^{2/\alpha}-\bar{z}^2(x,H=0),$$
(27)
where Γ is the Gamma function, $\bar{z}(x)\sim x^{2/\alpha}$ is centroid of concentration distribution. Considering the asymptotic results for $U(z)$ and $k_z(z)$ (given by (23)) following by

Monin-Obukhov similarity theory: free convection $(m=-1/3, n=4/3)$, neutral stratification $(m \approx 1/7, n=1)$ and strong ("z-less") stability $(m=1, n=0)$, we have:

$$\sigma_z(x) \sim \begin{cases} x^{3/2} & \text{at free convection} \\ x^{7/8} & \text{at neutral stratification} \\ x^{1/3} & \text{at strong stability.} \end{cases} \qquad (28)$$

In a identical way it can be determine the quantity skewness, which appear to be different from zero in correspondence with the non-Gaussian model (21), (24).

3. CONCLUSI0N

On the basis of power law approximation of the well known dispersion curves of Briggs in the frames of Gaussian and non-Gaussian Pollution model, it had been determined a series of main maximal and critical diffusion parameters and meteorological conditions at which they occur characterized by the Pasquill stability classes.

The majority of the results are given as exact analytical solutions which make them easy to use for estimation of the worst-case ambient conditions.

In result of taking into account the integral effect of wind turning in the non-Gaussian model it is easy to see that the maximal and critical diffusion parameters decrease compared to this determined with the Gaussian model in the other paragraphs.

A detail joint study of these effects together with the gravity deposition inversion effect is in interest of a future analysis.

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